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Compressive Sensing with Joint Measurement Matrix - Sparsifying Dictionary Optimization

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Abstract—Compressive sensing is the recent technique of data acquisition where perfect reconstruction of signal can be made from far fewer samples or measurement than traditional Shannon-Nyquist sampling theorem. In other words, compressive sensing provides a joint sensing and compression. Compressive sensing exploits the sparsity of signal that is signal can be represented with a few non-zero coefficients by using a suitable basis or sparsifying dictionary. In order to achieve good reconstruction performance from a few number of measurement, compressive sensing requires a small mutual coherence between a measurement matrix and the basis or dictionary. However, random matrix still can be optimized to achieve a smaller mutual coherence in order to improve the compressive sensing performance. This paper addresses the joint optimization between measurement matrix and sparsifying dictionary to minimize the average mutual coherence between them. Combination of KSVD and Gradient Descent method was used to perform this joint optimization. The joint optimized measurement matrix was used as the projection matrix for color image encoding to provide a compressive measurement. The simulation results showed that the joint optimization increases the PSNR and improve the visual perception of reconstructed image compared to the random matrix and optimized measurement matrix only, respectively.

I. INTRODUCTION

Compressive/Compressed Sensing, the term that was coined by Donoho in [1], is a novel signal acquisition method that offers joint sensing and compression. Hence, compressive sensing just need a small number of measurement to reconstruct the signal rather than conventional method that is Shannon-Nyquist sampling theorem [2,3]. Compressive Sensing becomes one of the recent interesting fields in signal and image processing communities after the important works of Donoho, Candès, Romberg and Tao in [1,4-6] that has initiated the research of compressive sensing. Compressive sensing is performed through a linear projection of the signal into a measurement matrix to yield a compressed measurement so that the process of measurement and compression is taken simultaneously. This is different from the traditional compression where the signal is sampled by following the Shannon-Nyquist sampling theorem and then compress it by using transform coding as in JPEG-2000 [7]. Compressive sensing utilizes the sparsity of natural signal that the signal can be represented with a few coefficients by using the appropriate basis such as Fourier, Wavelet, Curvelet or a dictionary. The basis or dictionary must be inconsistent with the measurement matrix so that the signal can be reconstructed from a few numbers of measurements [8].

Random matrix with high probability is inconsistent with many basis [6], however, as was showed in [9] and [10] the random matrix can be optimized by minimizing the coherence between the basis or dictionary that is used to represent the signal and result the better signal reconstruction. The optimizing methods in [9-10] also was applied in [11] and [12] to optimize the measurement matrix that result the better compressive sensing performance than the unoptimized one. Optimization method in [10] is better compared to the method in [9], however further improvement can be done by using learned dictionary. In [10] the optimized measurement matrix was applied to one dimension synthetic signal and gray image where symlet8 wavelet was used as sparsifying matrix. In this paper, further improvement of [10] was done by using KSVD method [13] for dictionary learning and simultaneously optimized the measurement matrix, yield joint measurement matrix – sparse dictionary optimization and extend the work of [10] to color image. The results in this paper showed that the joint optimization provide better compressive sensing performance than just optimizing the measurement matrix as in [10].

II. MEASUREMENT MATRIX OPTIMIZATION

Sparsity and incoherence play an important role in compressive sensing where they determine its performance [8]. A signal is said sparse if it has only a small number of nonzero components compared to its total length by using a proper basis or a dictionary. Compressive sensing framework mainly consists of two crucial parts: encoding (measurement/sensing) and decoding (reconstruction). Our focus in this paper is on the first part where we found that by joint optimizing the sensing process and sparse dictionary will more improve the reconstruction performance rather than only optimizing measurement matrix.

Consider a signal \( x \in \mathbb{R}^N \) and \( \Psi = [\psi_1 ... \psi_K] \in \mathbb{R}^{N \times K} \), where \( x \) can be written as:

\[
x = \sum_{k=1}^{K} \alpha_k \psi_k
\] where \( \alpha_k \) are the expansion coefficients, and if \( \| \alpha_k \|^2 = 1 \) and \( \psi_k \) is orthonormal, then the signal is sparse in \( \Psi \).

Therefore we can write the signal as:

\[
x = \Psi D \xi
\] where \( \xi \) is the vector containing the expansion coefficients of the signal in \( \Psi \) and \( D \) is the decomposition matrix. Hence, if the signal \( x \) is represented by a set of \( K \) distinct \( \psi_k \), it can be represented by \( K \) coefficients.

Sensing matrix \( \Psi \) is a matrix that maps \( x \) to \( y \) such that

\[
y = \Psi x
\] where \( y \) is a measurement that is smaller than the number of samples in \( x \).

Optimizing the sensing matrix to ensure that \( y \) is a good representation of the original signal \( x \) can be done by minimizing the coherence between the components of the signal and the columns of \( \Psi \). Coherence between the \( m \)-th column of \( \Psi \) and the \( n \)-th column of \( \Psi \) is given by

\[
\rho(m,n) = \frac{1}{||\psi_m||_2||\psi_n||_2} |\langle \psi_m, \psi_n \rangle|
\] so again minimizing

\[
\mu(\Psi) = \sum_{m,n} \rho(m,n)
\] will yield the best measurement matrix.

4. Conclusion

In this paper, we show that joint optimizing the sensing process and sparse dictionary will more improve the reconstruction performance rather than only optimizing measurement matrix.
\[ x = \Psi \theta \]  

where \( \theta = [\theta_1, ..., \theta_N]^T \) are coefficients vector that represent \( x \) in \( \Psi \). If \( \Psi \) is orthonormal and \( K = N \), it is called a basis and if \( K > N \) it is called over-complete dictionary and usually it is only called dictionary. Signal \( x \) in said \( S \)-sparse if \( \theta \) only has \( S \) non-zero coefficients and the others are zero. It is said compressible if the others coefficients have insignificant value compare to the \( S \)-significant coefficients. Sparse representation is used as the principal method in data compression as shown on JPEG-2000 [7].

Sensing matrix \( \Phi = [\Phi_1, ..., \Phi_m]^T \in \mathbb{R}^{m \times N} \) is used to encode then the signal \( x \) as a linear projection of it into \( \Phi \):

\[ y = \Phi x = \Phi \Psi \theta \]  

therefore \( y \) is a compressive measurement of \( x \) with length \( m << N \). The coherency of \( \Phi \) and \( \Psi \) is defined by:

\[ \mu(\Phi, \Psi) = \sqrt{N} \max_{1 \leq j \leq N} \sum_{i=1}^{m} \left| \langle \Phi_i, \Psi_j \rangle \right| \]  

where \( \mu(\Phi, \Psi) \in [1, \sqrt{N}] \), assume \( \Phi \) and \( \Psi \) are normalized. If the number of measurement fulfill:

\[ m \geq C \mu^2(\Phi, \Psi) S \log(N) \]  

where \( C \) is a positive constant and \( x \) is \( S \)-sparse, with overwhelming probability, \( x \) can be reconstructed from \( y \) exactly. If the support of \( \theta \) (number of nonzero coefficients) fulfill:

\[ \left\| \theta \right\|_0 < \frac{1}{2} \left( 1 + \frac{1}{(\mu(\Phi, \Psi)^2)} \right) \]  

so again \( x \) can be reconstructed from \( y \) exactly. In (4) and (5) show that Compressive sensing requires a small value of \( \mu(\Phi, \Psi) \) in order to just needs a few number of measurements and the sparsity condition is not too tight.

A. Optimized Measurement Matrix

In (2), \( \Phi \Psi = D \), where \( D \) is called equivalent dictionary, in [9], Elad proposed the mutual coherence parameter, \( \mu(D) \) that is defined by:

\[ \mu(D) = \max_{j \neq i, j \neq k} \left| \langle d_i, d_k \rangle \right| \]  

that is related to the coherency of \( \Phi \) and \( \Psi \), assume the column of \( D \) has a unit 2-norm . The sensing matrix can be optimized by minimizing \( \mu(D) \), it is done by making the Gram matrix of \( D \), \( G = D^TD \) as close as identity matrix \( I \):

\[ D = \arg \min_D \left\| D^TD - I \right\|_F^2 \]  

Vahid Abolghasemi et al. in [10] proposed to minimize the mutual coherence of equivalent dictionary by using Gradient Descent method. First by defining the corresponding error as:

\[ E = \left\| D^TD - I \right\|_F^2 = \text{Tr} \left( (D^TD - I)(D^TD - I)^T \right) \]  

then compute the gradient of \( E \) with respect to elements of \( D \) that is \( d_{i,j} \):

\[ \frac{\partial E}{\partial d_{i,j}} = -4D_{i,j} \]  

using (9), the solution of (7) can be described as an iterative process to update \( D \) by using:

\[ D_{(i+1)} = D_{(i)} - \eta D_{(i)}(D_{(i)}D_{(i)} - I) \]  

where \( \eta \) is the step size. The updated sensing matrix can be achieved by using \( \Phi = D^\Psi^{-1} \) and before run to the next iteration step, \( D \) must be normalized again so that the column has a unit 2-norm. After a number of iterations the solution of (7) can be achieved and we get the optimized sensing matrix.

B. KSVD - Gradient Descent Method

The KSVD algorithm was used in [13] to learn the sparse dictionary by solving the following optimization problem:

\[ \min_{\Psi, \theta} \left\| X - \Psi \theta \right\|_2^2, \text{s.t. } \left\| \theta \right\|_0 \leq S, i = 1, ..., P \]  

where \( X \in \mathbb{R}^{N \times P} \) is a set of \( P \) training patch, \( \Psi \in \mathbb{R}^{N \times K} \) and \( \theta \in \mathbb{R}^{K \times P} \). The K-SVD algorithm obtains the dictionary update by separate SVD computations for each column of \( \Psi \) sequentially, which explains its name. The KSVD - Gradient Descent method that we proposed in this paper was carried out like the following : initially, a certain of \( \Psi \) and \( \Phi \) are used, measurement matrix \( \Phi \) is optimized by using Gradient Descent method. The OMP (Orthogonal Matching Pursuit) [14] is used to obtain \( \Phi \) from (11). Next, the updated dictionary \( \Psi \) is found by using KSVD method where this learned dictionary will be used in the next iteration to optimize the measurement matrix \( \Phi \) and KSVD method is used again to get the learned dictionary \( \Phi \), the process is repeated for a number of iteration or until the stopping criteria is attained.
III. SIMULATION METHOD

We used 50 training-images (available in [13]) as shown on
Fig. 1, where YCbCr models was used as color space format.
From each image and each color component of YCbCr, it is
taken randomly 300 patches (8 x 8 pixels), so we have 15000
patches for each color component of YCbCr. The training
patches were used in Gradient Descent optimization and
KSVD method to obtain the optimized measurement matrix
and the learned sparsifying dictionary. Joint KSVD-Gradient
Descent optimization method was performed separately for
each color component of YCbCr.

We used three different scenarios of compressive sensing:

1. KSVD-Random: Random matrix is used as measurement
   matrix and KSVD algorithm is used to get sparsifying
dictionary.
2. Uncoupled KSVD-Gradient Descent: Random matrix is
   optimized by using Gradient Descent method and KSVD
   algorithm is used to get the sparsifying dictionary, both
   them were performed separately.
3. Joint KSVD-Gradient Descent: Joint Measurement
   Matrix - Sparsifying Dictionary Optimization was
   performed by using KSVD-Gradient Descent where
   random matrix is used initially.

We used overcomplete DCT as initial dictionary \( \Psi \) with
\( K = 4 N = 256 \). We used a test image to perform those
scenarios and in order to reduce the computational complexity
in compressive sensing reconstruction, the image was divided
into 8 x 8 non-overlapping blocks. The next step is to convert
each block to 1-D signal, so \( N = 64 \), compressive
measurement was performed on those blocks by using \( \Phi \)
from one of the three scenarios above to yield \( Y \in \mathbb{R}^{m \times 64} \).
The random matrix \( \Phi \in \mathbb{R}^{m \times 64} \) was generated by \( \frac{1}{\sqrt{m}} \)
Gaussian elements and normalized by \( \sqrt{m} \) where \( m \) is a
number of measurement that was varied from 8 to 32. We used
\( \eta = 0.01 \) and ten iterations for Gradient Descent optimization.
The reconstruction of each block was performed by using
OMP to get \( \Theta \) from \( Y \) and reconstructed blocks by using
\( X = \Psi \Theta \) are obtained. Finally we deblocked the whole
reconstructed blocks to get the reconstructed image. We used
the Peak Signal-to-Noise Ratio (PSNR) in Decibel (dB) to
measure the reconstruction performance. The PSNR is defined
in (12), where \( MSE \) is Mean Square Error of reconstructed
image, \( Max_i = 1 \) is the maximum possible pixel value of
the image, \( W \) and \( H \) are width and height of image.

\[
PSNR[dB] = 10 \log_{10} \left( \frac{Max_i^2}{MSE} \right) = 10 \log_{10} \left( \frac{W \cdot H \cdot 12}{\sum_{x=1}^{W} \sum_{y=1}^{H} (\text{reconstructed}_x - \text{test}_x)^2} \right)
\]  

IV. RESULTS & DISCUSSION

We used image size \( W \times H = 321 \times 481 \) as a test image as shown in Fig. 2.

![Figure 2. Test image to perform the three different scenarios of compressive sensing.](image)

Fig. 3 and Fig 4. show the PSNR of reconstructed image
from compressive sensing by using OMP and Iteratively
Reweighted Least Squares (IRLS) - \( \ell_p \) - minimization [16]
with \( p = 0.8 \) as a function of Ratio Measurement Numbers
(RMN). RMN is ratio between number of compressive
measurement and total pixel of test image patches for the three
different scenarios above. We calculated the average PSNR of
all reconstructed YCbCr color component. It show for RMN =
12.5 % to 50 % by optimizing the measurement matrix can
improve the Gradient Descent. 

![Figure 3. The sensing by us Descend and Jo](image)

![Figure 4. The sensing by us OMP - Gradient Descend up to 13 % Descend, be increment compared 10 % comp RMN = 31 Fig. 5 s sensing of 25%. It sh Fig. 5.b reconstruct Joint KSI Uncoupled Fig. 6 s sensing ca It also pro](image)
impressive using $\Phi \in \mathbb{R}^{m \times n}$, with $i.i.d. \geq m$ is a sample. We used imitator by using blocks by the whole. We used $\ell_p$ ($P \leq 2.0$) to is defined constructed hue of the image as

![Graph](image)

Figure 3: The PSNR comparison of reconstructed image from compressive sensing by using OMP for: KSVD-Random, Uncoupled KSVD-Gradient Descent and Joint KSVD-Gradient Descent.

![Image](image)

Figure 4: The PSNR comparison of reconstructed image from compressive sensing by using (IRLS) - $\ell_p$ - minimization for: KSVD-Random, Uncoupled KSVD-Gradient Descent and Joint KSVD-Gradient Descent.

OMP provides PSNR increment of Joint KSVD-Gradient Descent up to 215% compared to the KSVD-Random and up to 13% at compared to the Uncoupled KSVD-Gradient Descent, both at RMN = 12.5%. IRLS - $\ell_p$ provides the PSNR increment of Joint KSVD-Gradient Descent up to 151% compared to the KSVD-Random at RMN = 12.5% and up to 10% compared to the Uncoupled KSVD-Gradient Descent at RMN = 31.25%.

![Image](image)

Figure 5: The comparison of reconstructed image for RMN = 25% by using OMP: (a) KSVD-Random, (b) Uncoupled KSVD-Gradient Descent, (c) Joint KSVD-Gradient Descent.

We also performed compressive measurement of noised image by adding white Gaussian noise with zero mean and variance $= 0.01$, where by using overlapping-blocks can reduce the noise of reconstructed image than by using non-overlapping blocks as shown on Fig. 7.

![Image](image)

Figure 6: The PSNR comparison of reconstructed image from compressive sensing of the three scenarios above by using OMP at RMN = 25%. It shows clearly that by optimizing measurement matrix (Fig 5.b and Fig 5.c) improve the visual perception of reconstructed image compared to random matrix (Fig 5.a) and Joint KSVD-Gradient Descent is slightly better than Uncoupled KSVD-Gradient Descent about 0.74 dB.

![Image](image)

(a) $PSNR = 15.58dB$
(b) $PSNR = 26.88dB$

(c) $PSNR = 27.62dB$

Figure 7: The comparison of reconstructed image for RMN = 25% by using OMP: (a) KSVD-Random, (b) Uncoupled KSVD-Gradient Descent, (c) Joint KSVD-Gradient Descent.
Figure 6. The comparison of reconstructed image for RMN = 25 % by using overlapping-blocks where: (a) KSVD-Random, Uncoupled KSVD-Gradient Descent and Joint KSVD-Gradient Descent.

(a) PSNR = 25.51 dB  
(b) PSNR = 21.77 dB  
(c) PSNR = 31.86 dB

Figure 7. The reconstructed image by using OMP from compressive measurement of (a) noisy image for RMN = 25 % by using Joint KSVD-Gradient Descent, where: (b) non-overlapping blocks (c) overlapping-blocks.

V. CONCLUSION

In this paper, we proposed joint KSVD and Gradient Descent method to improve the reconstruction image performance from compressive sensing. Based on the results, it showed that by using the joint optimization improved the reconstructed image compared to the random matrix and optimized measurement matrix only. Furthermore, by using overlapping-blocks, the reconstructed image from the compressive measurement of noisy image can be improved. Further improvement can be achieved in future work by optimizing measurement matrix and dictionary learning simultaneously based on block-sparse representations.

REFERENCES


